https://workshop.f4sg.org/africast/

Africast-Time Series Analysis & Forecasting Using R

7. Exponential smoothing

tsibble feasts fable

Outline

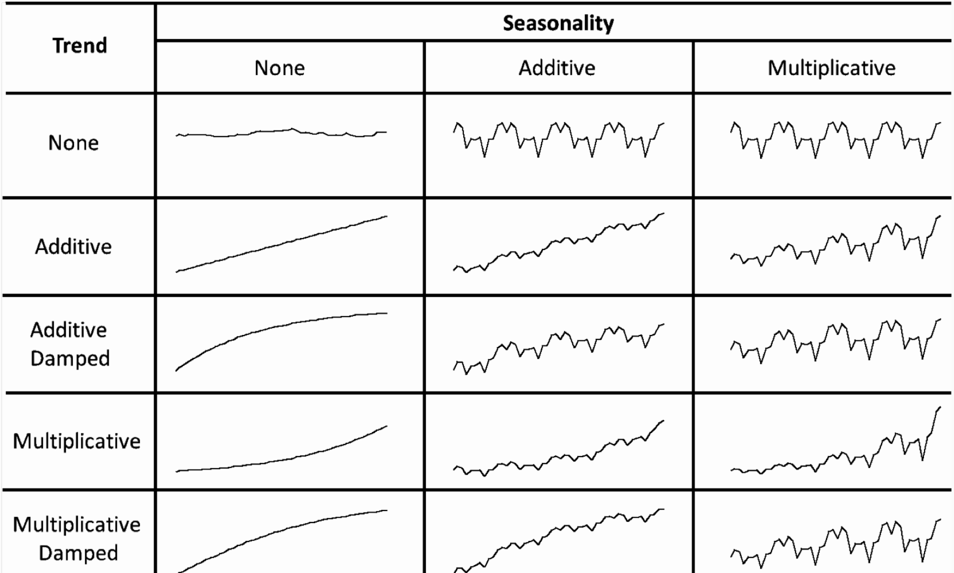
1. Exponential smoothing
2. Trend methods
3. Seasonal methods
4. ETS taxonomy
5. Non-Gaussian forecast distributions

Outline

From simple methods to Exponential Smoothing

- Naive method: Use only the last observation
- Average method: Use all observations
- Want something in between naive and average methods.
- Most recent data should have more weight.
- This is exactly the concept behind exponential smoothing

Pegel's classification



Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a "level", "trend" (slope) and "seasonal" component to describe a time series.
- The rate of change of the components are controlled by "smoothing parameters": α, β and γ respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

A model for levels, trends, and seasonalities

We want a model that captures the level (ℓ_t), trend (b_t) and seasonality (s_t).

How do we combine these elements?

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Additively?

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Multiplicatively?

$$y_t = \ell_{t-1} b_{t-1} s_{t-m}(1 + \varepsilon_t)$$

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Perhaps a mix of both?

$$y_t = (\ell_{t-1} + b_{t-1}) s_{t-m} + \varepsilon_t$$

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Perhaps a mix of both?

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How do the level, trend and seasonal components evolve over time?

ETS models

General notation E T S : **E**xponen**T**ial **S**moothing

↗ ↑ ↖

Error **T**rend **S**eason

Error: Additive (`"A"`) or multiplicative (`"M"`)

ETS models

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Error: Additive (`"A"`) or multiplicative (`"M"`)

Trend: None (`"N"`), additive (`"A"`), multiplicative (`"M"`), or damped (`"Ad"` or `"Md"`).

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Error: Additive (`"A"`) or multiplicative (`"M"`)

Trend: None (`"N"`), additive (`"A"`), multiplicative (`"M"`), or damped (`"Ad"` or `"Md"`).

Seasonality: None (`"N"`), additive (`"A"`) or multiplicative (`"M"`)

ETS(A,N,N): SES with additive errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1} + \varepsilon_t$
State equation	$\ell_t = \ell_{t-1} + \alpha\varepsilon_t$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

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where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

- "innovations" or "single source of error" because equations have the same error process, ε_t.
- Measurement equation: relationship between observations and states.
- Transition/state equation(s): evolution of state(s) over time.

ETS(M,N,N): SES with multiplicative errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1}(1 + \varepsilon_t)$
State equation	$\ell_t = \ell_{t-1}(1 + \alpha\varepsilon_t)$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

ETS(M,N,N): SES with multiplicative errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T$
Measurement equation	$y_t = \ell_{t-1}(1 + \varepsilon_t)$
State equation	$\ell_t = \ell_{t-1}(1 + \alpha\varepsilon_t)$

where $\varepsilon_t \sim \text{NID}(0, \sigma^2)$.

- Models with additive and multiplicative errors with the same parameters generate the same point forecasts but different prediction intervals.

Outline

Holt's linear trend

Additive errors: ETS(A,A,N)

Forecast equation $\hat{y}_{T+h|T} = \ell_T + hb_T$

Measurement equation $y_t = \ell_{t-1} + b_{t-1} + \varepsilon_t$

State equations
$$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$$
$$b_t = b_{t-1} + \beta\varepsilon_t$$

Holt's linear trend

Additive errors: ETS(A,A,N)

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T + hb_T$
Measurement equation	$y_t = \ell_{t-1} + b_{t-1} + \varepsilon_t$
State equations	$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$
	$b_t = b_{t-1} + \beta\varepsilon_t$

Multiplicative errors: ETS(M,A,N)

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T + hb_T$
Measurement equation	$y_t = (\ell_{t-1} + b_{t-1})(1 + \varepsilon_t)$
State equations	$\ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t)$
	$b_t = b_{t-1} + \beta(\ell_{t-1} + b_{t-1})\varepsilon_t$

Example: Australian population

```
aus_economy <- global_economy |>
  filter(Country == "Australia") |>
  mutate(Pop = Population / 1e6)
fit <- aus_economy |> model(AAN = ETS(Pop))
report(fit)
```

```
Series: Pop
Model: ETS(A,A,N)
  Smoothing parameters:
    alpha = 1
    beta  = 0.327

  Initial states:
 l[0]  b[0]
 10.1 0.222

  sigma^2:  0.0041

  AIC  AICc   BIC
 77.0  75.8  66.7
```

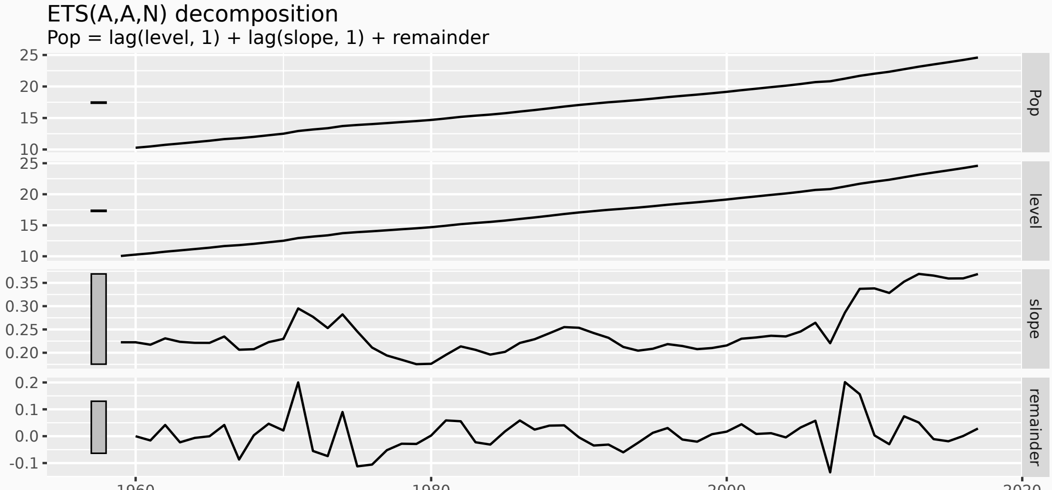
Example: Australian population

```
components(fit)
```

```
# A dable: 59 x 7 [1Y]
# Key:      Country, .model [1]
# :         Pop = lag(level, 1) + lag(slope, 1) + remainder
   Country   .model  Year   Pop level slope remainder
   <fct>     <chr>  <dbl> <dbl> <dbl> <dbl>     <dbl>
 1 Australia AAN     1959  NA    10.1 0.222 NA
 2 Australia AAN     1960  10.3  10.3 0.222 -0.000145
 3 Australia AAN     1961  10.5  10.5 0.217 -0.0159
 4 Australia AAN     1962  10.7  10.7 0.231  0.0418
 5 Australia AAN     1963  11.0  11.0 0.223 -0.0229
 6 Australia AAN     1964  11.2  11.2 0.221 -0.00641
 7 Australia AAN     1965  11.4  11.4 0.221 -0.000314
 8 Australia AAN     1966  11.7  11.7 0.235  0.0418
 9 Australia AAN     1967  11.8  11.8 0.206 -0.0869
10 Australia AAN     1968  12.0  12.0 0.208  0.00350
```

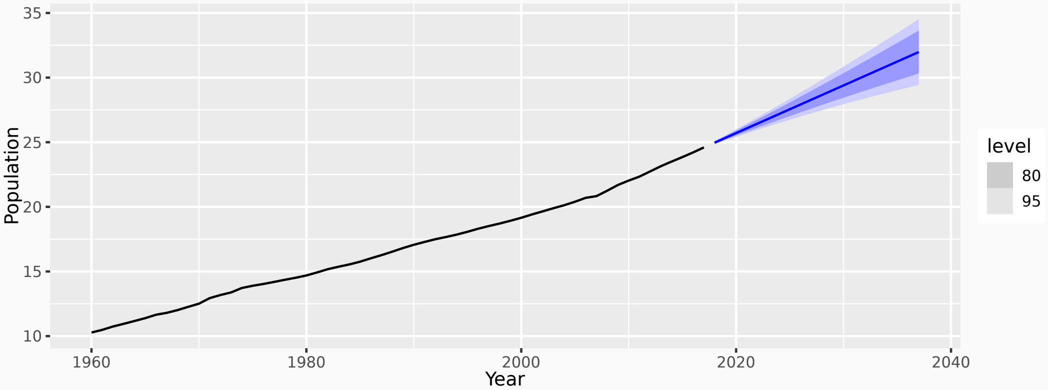
Example: Australian population

```r
components(fit) |> autoplot()
```



Example: Australian population

```
fit |>
  forecast(h = 20) |>
  autoplot(aus_economy) +
  labs(y = "Population", x = "Year")
```



ETS(A,Ad,N): Damped trend method

Additive errors

Forecast equation $\hat{y}_{T+h|T} = \ell_T + (\phi + \cdots + \phi^{h-1})b_T$

Measurement equation $y_t = (\ell_{t-1} + \phi b_{t-1}) + \varepsilon_t$

State equations
$$\ell_t = (\ell_{t-1} + \phi b_{t-1}) + \alpha\varepsilon_t$$
$$b_t = \phi b_{t-1} + \beta\varepsilon_t$$

ETS(A,Ad,N): Damped trend method

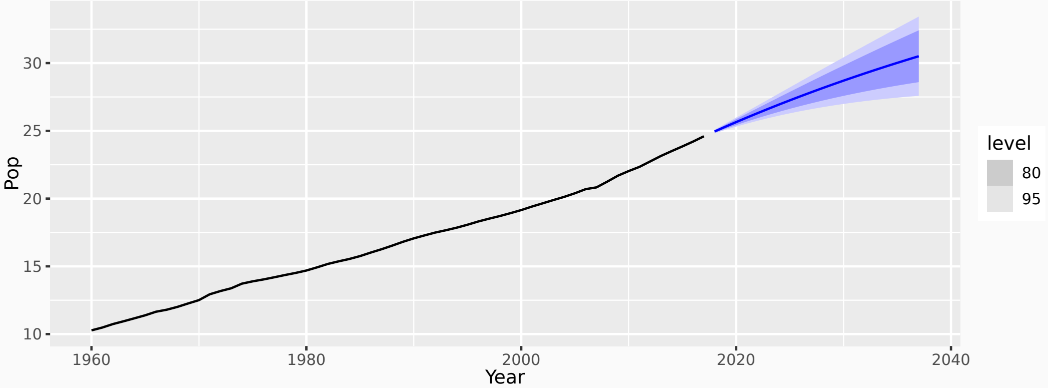
Additive errors

Forecast equation	$\hat{y}_{T+h\|T} = \ell_T + (\phi + \cdots + \phi^{h-1})b_T$
Measurement equation	$y_t = (\ell_{t-1} + \phi b_{t-1}) + \varepsilon_t$
State equations	$\ell_t = (\ell_{t-1} + \phi b_{t-1}) + \alpha\varepsilon_t$
	$b_t = \phi b_{t-1} + \beta\varepsilon_t$

- Damping parameter $0 < \phi < 1$.
- If $\phi = 1$, identical to Holt's linear trend.
- As $h \to \infty$, $\hat{y}_{T+h|T} \to \ell_T + \phi b_T/(1 - \phi)$.
- Short-run forecasts trended, long-run forecasts constant.

Example: Australian population

```r
aus_economy |>
  model(holt = ETS(Pop ~ trend("Ad"))) |>
  forecast(h = 20) |>
  autoplot(aus_economy)
```



Example: National populations

```r
fit <- global_economy |>
  mutate(Pop = Population / 1e6) |>
  model(ets = ETS(Pop))
fit
```

```
# A mable: 263 x 2
# Key:     Country [263]
   Country                       ets
   <fct>                     <model>
 1 Afghanistan          <ETS(A,A,N)>
 2 Albania              <ETS(M,A,N)>
 3 Algeria              <ETS(M,A,N)>
 4 American Samoa       <ETS(M,A,N)>
 5 Andorra              <ETS(M,A,N)>
 6 Angola               <ETS(M,A,N)>
 7 Antigua and Barbuda <ETS(M,A,N)>
 8 Arab World           <ETS(M,A,N)>
 9 Argentina            <ETS(A,A,N)>
10 Armenia              <ETS(M,A,N)>
# i 253 more rows
```

Example: National populations

```
fit |>
  forecast(h = 5)
```

```
# A fable: 1,315 x 5 [1Y]
# Key:     Country, .model [263]
   Country     .model  Year              Pop .mean
   <fct>       <chr>  <dbl>           <dist> <dbl>
 1 Afghanistan ets     2018    N(36, 0.012) 36.4 
 2 Afghanistan ets     2019    N(37, 0.059) 37.3 
 3 Afghanistan ets     2020     N(38, 0.16) 38.2 
 4 Afghanistan ets     2021     N(39, 0.35) 39.0 
 5 Afghanistan ets     2022     N(40, 0.64) 39.9 
 6 Albania     ets     2018 N(2.9, 0.00012)  2.87
 7 Albania     ets     2019   N(2.9, 6e-04)  2.87
 8 Albania     ets     2020  N(2.9, 0.0017)  2.87
 9 Albania     ets     2021  N(2.9, 0.0036)  2.86
10 Albania     ets     2022  N(2.9, 0.0066)  2.86
# i 1,305 more rows
```

Outline

ETS(A,A,A): Holt-Winters additive method

Forecast equation	$\hat{y}_{t+h\|t} = \ell_t + hb_t + s_{t+h-m(k+1)}$
Observation equation	$y_t = \ell_{t-1} + b_{t-1} + s_{t-m} + \varepsilon_t$
State equations	$\ell_t = \ell_{t-1} + b_{t-1} + \alpha\varepsilon_t$
	$b_t = b_{t-1} + \beta\varepsilon_t$
	$s_t = s_{t-m} + \gamma\varepsilon_t$

- $k =$ integer part of $(h-1)/m$.
- $\sum_i s_i \approx 0$.
- Parameters: $0 \le \alpha \le 1,\ 0 \le \beta^* \le 1,\ 0 \le \gamma \le 1-\alpha$ and $m =$ period of seasonality (e.g. $m = 4$ for quarterly data).

ETS(M,A,M): Holt-Winters multiplicative method

Forecast equation	$\hat{y}_{t+h\|t} = (\ell_t + hb_t)s_{t+h-m(k+1)}$
Observation equation	$y_t = (\ell_{t-1} + b_{t-1})s_{t-m}(1 + \varepsilon_t)$
State equations	$\ell_t = (\ell_{t-1} + b_{t-1})(1 + \alpha\varepsilon_t)$
	$b_t = b_{t-1}(1 + \beta\varepsilon_t)$
	$s_t = s_{t-m}(1 + \gamma\varepsilon_t)$

- k is integer part of $(h-1)/m$.
- $\sum_i s_i \approx m$.
- Parameters: $0 \le \alpha \le 1,\ 0 \le \beta^* \le 1,\ 0 \le \gamma \le 1-\alpha$ and $m =$ period of seasonality (e.g. $m = 4$ for quarterly data).

Example: Australian holiday tourism

```
holidays <- tourism |>
  filter(Purpose == "Holiday")
fit <- holidays |> model(ets = ETS(Trips))
fit
```

```
# A mable: 76 x 4
# Key:     Region, State, Purpose [76]
   Region          State Purpose           ets
   <chr>           <chr> <chr>         <model>
 1 Adelaide        SA    Holiday <ETS(A,N,A)>
 2 Adelaide Hills  SA    Holiday <ETS(A,A,N)>
 3 Alice Springs   NT    Holiday <ETS(M,N,A)>
 4 Ballarat        VIC   Holiday <ETS(M,N,A)>
 5 Barkly          NT    Holiday <ETS(A,N,A)>
 6 Barossa         SA    Holiday <ETS(A,N,N)>
 7 Bendigo Loddon  VIC   Holiday <ETS(M,N,N)>
 8 Blue Mountains  NSW   Holiday <ETS(M,N,M)>
 9 Brisbane        QLD   Holiday <ETS(A,A,N)>
```

Example: Australian holiday tourism

```
fit |>
  filter(Region == "Snowy Mountains") |>
  report()
```

```
Series: Trips
Model: ETS(M,N,A)
  Smoothing parameters:
    alpha = 0.157
    gamma = 1e-04

  Initial states:
 l[0] s[0] s[-1] s[-2] s[-3]
  142  -61   131 -42.2 -27.7

  sigma^2:  0.0388

 AIC AICc  BIC
 852  854  869
```

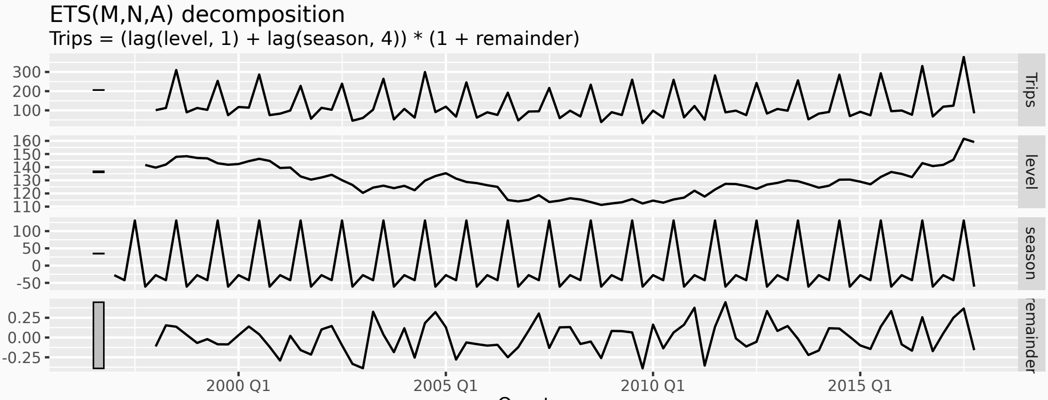
Example: Australian holiday tourism

```
fit |>
  filter(Region == "Snowy Mountains") |>
  components(fit)
```

```
# A dable: 84 x 9 [1Q]
# Key:     Region, State, Purpose, .model [1]
# :        Trips = (lag(level, 1) + lag(season, 4)) * (1 + remainder)
   Region          State Purpose .model Quarter Trips level season remainder
   <chr>           <chr> <chr>   <chr>    <qtr> <dbl> <dbl>  <dbl>     <dbl>
 1 Snowy Mountains NSW   Holiday ets    1997 Q1  NA      NA  -27.7    NA
 2 Snowy Mountains NSW   Holiday ets    1997 Q2  NA      NA  -42.2    NA
 3 Snowy Mountains NSW   Holiday ets    1997 Q3  NA      NA  131.     NA
 4 Snowy Mountains NSW   Holiday ets    1997 Q4  NA     142. -61.0    NA
 5 Snowy Mountains NSW   Holiday ets    1998 Q1 101.    140. -27.7    -0.113
 6 Snowy Mountains NSW   Holiday ets    1998 Q2 112.    142. -42.2     0.154
 7 Snowy Mountains NSW   Holiday ets    1998 Q3 310.    148. 131.      0.137
 8 Snowy Mountains NSW   Holiday ets    1998 Q4  89.8  148. -61.0     0.0335
 9 Snowy Mountains NSW   Holiday ets    1999 Q1 112.    147. -27.7    -0.0687
```

Example: Australian holiday tourism

```
fit |>
  filter(Region == "Snowy Mountains") |>
  components(fit) |>
  autoplot()
```



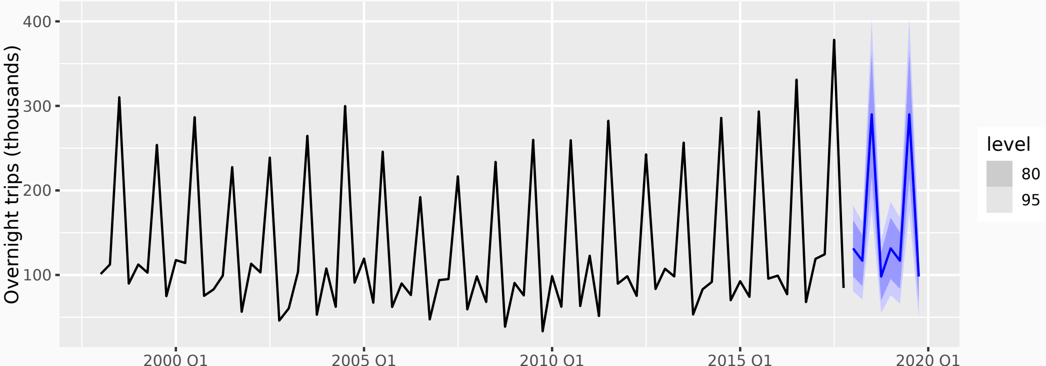
Example: Australian holiday tourism

```
fit |> forecast()
```

```
# A fable: 608 x 7 [1Q]
# Key:     Region, State, Purpose, .model [76]
   Region          State Purpose .model Quarter        Trips .mean
   <chr>           <chr> <chr>   <chr>    <qtr>       <dist> <dbl>
 1 Adelaide        SA    Holiday ets    2018 Q1 N(210, 457) 210.
 2 Adelaide        SA    Holiday ets    2018 Q2 N(173, 473) 173.
 3 Adelaide        SA    Holiday ets    2018 Q3 N(169, 489) 169.
 4 Adelaide        SA    Holiday ets    2018 Q4 N(186, 505) 186.
 5 Adelaide        SA    Holiday ets    2019 Q1 N(210, 521) 210.
 6 Adelaide        SA    Holiday ets    2019 Q2 N(173, 537) 173.
 7 Adelaide        SA    Holiday ets    2019 Q3 N(169, 553) 169.
 8 Adelaide        SA    Holiday ets    2019 Q4 N(186, 569) 186.
 9 Adelaide Hills SA    Holiday ets    2018 Q1   N(19, 36)  19.4
10 Adelaide Hills SA    Holiday ets    2018 Q2   N(20, 36)  19.6
# i 598 more rows
```

Example: Australian holiday tourism

```
fit |>
  forecast() |>
  filter(Region == "Snowy Mountains") |>
  autoplot(holidays) +
  labs(x = "Year", y = "Overnight trips (thousands)")
```



Outline

Exponential smoothing models

Additive Error

Trend Component		**Seasonal Component** N (None)	A (Additive)	M (Multiplicative)
N	(None)	A,N,N	A,N,A	~~A,N,M~~
A	(Additive)	A,A,N	A,A,A	~~A,A,M~~
A_d	(Additive damped)	A,A_d,N	A,A_d,A	~~A,A_d,M~~

Multiplicative Error

Trend Component		**Seasonal Component** N (None)	A (Additive)	M (Multiplicative)
N	(None)	M,N,N	M,N,A	M,N,M
A	(Additive)	M,A,N	M,A,A	M,A,M
A_d	(Additive damped)	M,A_d,N	M,A_d,A	M,A_d,M

Estimating ETS models

- Smoothing parameters α, β, γ and ϕ, and the initial states ℓ_0, b_0, $s_0, s_{-1}, \dots, s_{-m+1}$ are estimated by maximising the "likelihood" = the probability of the data arising from the specified model.
- For models with additive errors equivalent to minimising SSE.
- For models with multiplicative errors, **not** equivalent to minimising SSE.

Model selection

Akaike's Information Criterion

$$\text{AIC} = -2\log(\text{L}) + 2k$$

where L is the likelihood and k is the number of parameters & initial states estimated in the model.

Model selection

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$$\text{AIC} = -2\log(\text{L}) + 2k$$

where L is the likelihood and k is the number of parameters & initial states estimated in the model.

Corrected AIC

$$\text{AIC}_\text{c} = \text{AIC} + \frac{2k(k+1)}{T-k-1}$$

which is the AIC corrected (for small sample bias).

Model selection

Akaike's Information Criterion

$$\text{AIC} = -2\log(\text{L}) + 2k$$

where L is the likelihood and k is the number of parameters & initial states estimated in the model.

Corrected AIC

$$\text{AIC}_{\text{c}} = \text{AIC} + \frac{2k(k+1)}{T-k-1}$$

which is the AIC corrected (for small sample bias).

Bayesian Information Criterion

$$\text{BIC} = \text{AIC} + k(\log(T) - 2).$$

AIC and cross-validation

Minimizing the AIC assuming Gaussian residuals is asymptotically equivalent to minimizing one-step time series cross validation MSE.

Automatic forecasting

From Hyndman et al. (IJF, 2002):

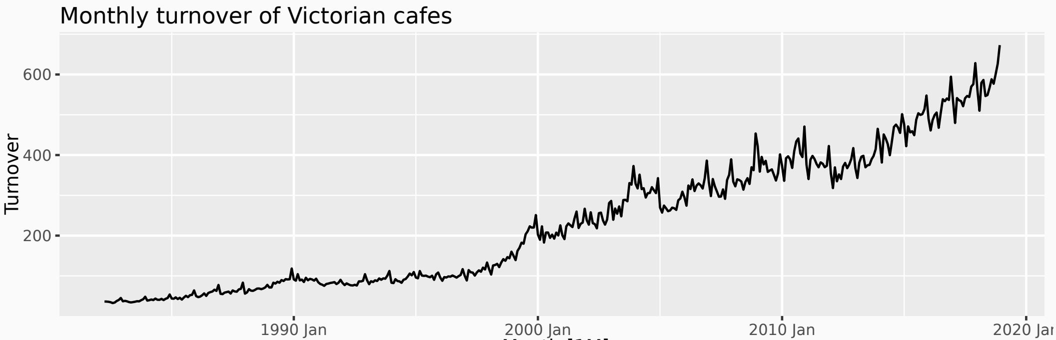
1. Apply each model that is appropriate to the data. Optimize parameters and initial values using MLE.
2. Select best method using AICc.
3. Produce forecasts using best method.
4. Obtain forecast intervals using underlying state space model.

- Method performed very well in M3 competition.
- Used as a benchmark in the M4 competition.

Outline

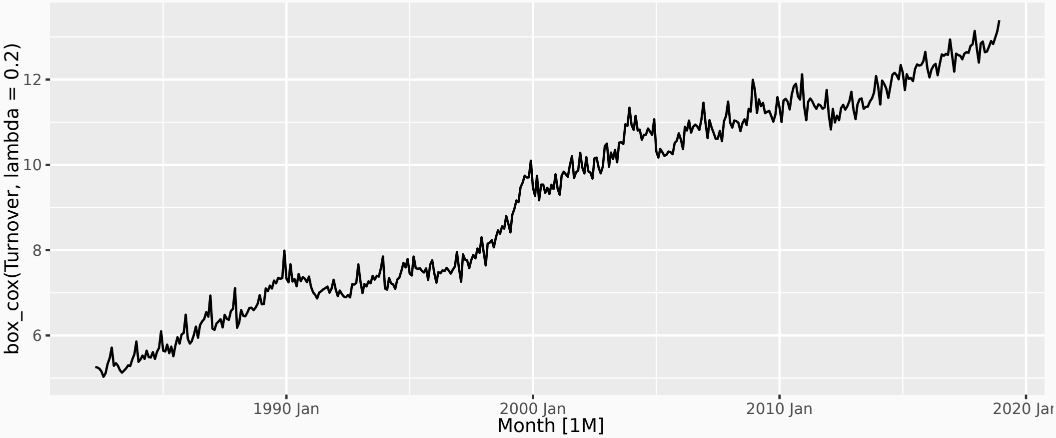
Non-Gaussian forecast distributions

```r
vic_cafe <- tsibbledata::aus_retail |>
  filter(State == "Victoria",
         Industry == "Cafes, restaurants and catering services") |>
  select(Month, Turnover)
vic_cafe |>
  autoplot(Turnover) + labs(title = "Monthly turnover of Victorian cafes")
```



Forecasting with transformations

```
vic_cafe |> autoplot(box_cox(Turnover, lambda = 0.2))
```



Forecasting with transformations

```
fit <- vic_cafe |>
  model(ets = ETS(box_cox(Turnover, 0.2)))
fit
```

```
# A mable: 1 x 1
           ets
       <model>
1 <ETS(A,A,A)>
```

```
(fc <- fit |> forecast(h = "3 years"))
```

```
# A fable: 36 x 4 [1M]
# Key:     .model [1]
   .model     Month        Turnover .mean
   <chr>      <mth>          <dist> <dbl>
 1 ets     2019 Jan  t(N(13, 0.02))  608.
 2 ets     2019 Feb t(N(13, 0.028))  563.
 3 ets     2019 Mar t(N(13, 0.036))  629.
 4 ets     2019 Apr t(N(13, 0.044))  615.
```

Forecasting with transformations

```
fit <- vic_cafe |>
  model(ets = ETS(box_cox(Turnover, 0.2)))
fit
```

```
# A mable: 1 x 1
            ets
        <model>
1 <ETS(A,A,A)>
```

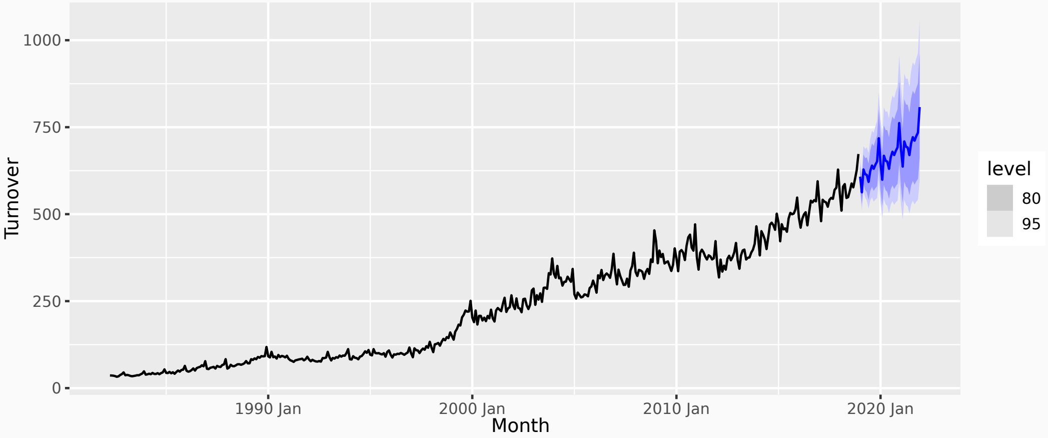
```
(fc <- fit |> forecast(h = "3 years"))
```

```
# A fable: 36 x 4 [1M]
# Key:     .model [1]
   .model     Month          Turnover .mean
   <chr>      <mth>            <dist> <dbl>
 1 ets     2019 Jan  t(N(13, 0.02))  608.
 2 ets     2019 Feb t(N(13, 0.028))  563.
 3 ets     2019 Mar t(N(13, 0.036))  629.
 4 ets     2019 Apr t(N(13, 0.044))  615.
```

- `t(N)` denotes a transformed normal distribution.
- back-transformation and bias adjustment is done automatically.

Forecasting with transformations

```
fc |> autoplot(vic_cafe)
```



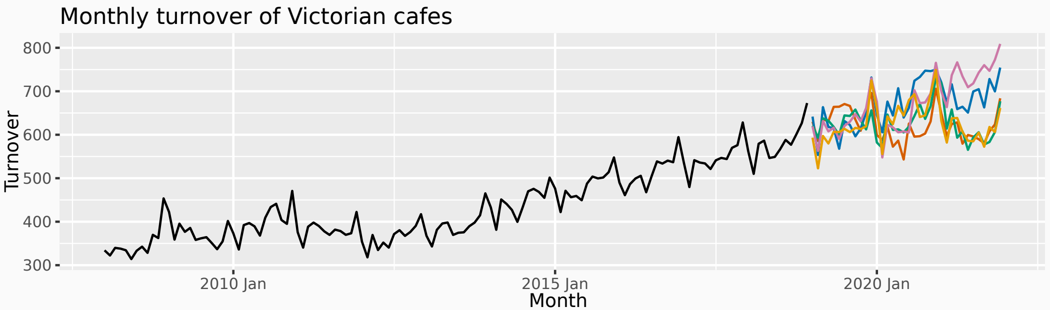
Bootstrapped forecast distributions

```r
sim <- fit |> generate(h = "3 years", times = 5, bootstrap = TRUE)
sim
```

```
# A tsibble: 180 x 5 [1M]
# Key:       .model, .rep [5]
   .model .rep     Month  .innov  .sim
   <chr>  <chr>    <mth>   <dbl> <dbl>
 1 ets    1     2019 Jan  0.132   630.
 2 ets    1     2019 Feb  0.0635  586.
 3 ets    1     2019 Mar -0.0811  635.
 4 ets    1     2019 Apr  0.0273  631.
 5 ets    1     2019 May  0.209   664.
 6 ets    1     2019 Jun  0.199   664.
 7 ets    1     2019 Jul -0.0763  671.
 8 ets    1     2019 Aug -0.143   666.
 9 ets    1     2019 Sep -0.175   635.
10 ets    1     2019 Oct -0.279   609.
# i 170 more rows
```

Bootstrapped forecast distributions

```
vic_cafe |>
  filter(year(Month) >= 2008) |>
  ggplot(aes(x = Month)) +
  geom_line(aes(y = Turnover)) +
  geom_line(aes(y = .sim, colour = as.factor(.rep)), data = sim) +
  labs(title = "Monthly turnover of Victorian cafes") +
  guides(col = FALSE)
```



Bootstrapped forecast distributions

```
fc <- fit |> forecast(h = "3 years", bootstrap = TRUE)
fc
```

```
# A fable: 36 x 4 [1M]
# Key:     .model [1]
   .model     Month      Turnover .mean
   <chr>      <mth>        <dist> <dbl>
 1 ets     2019 Jan sample[5000]   608.
 2 ets     2019 Feb sample[5000]   563.
 3 ets     2019 Mar sample[5000]   629.
 4 ets     2019 Apr sample[5000]   615.
 5 ets     2019 May sample[5000]   613.
 6 ets     2019 Jun sample[5000]   593.
 7 ets     2019 Jul sample[5000]   624.
 8 ets     2019 Aug sample[5000]   640.
 9 ets     2019 Sep sample[5000]   631.
10 ets     2019 Oct sample[5000]   643.
# i 26 more rows
```

Bootstrapped forecast distributions

```r
fc |> autoplot(vic_cafe) +
  labs(title = "Monthly turnover of Victorian cafes")
```